UG-04-04
hep-th/0407047

Scalar Cosmology with Multi-exponential Potentials

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Abstract

We investigate cosmologies with an arbitrary number of scalars and the most general multi-exponential potential. By formulating the equations of motion in terms of autonomous systems we complete the classification of power-law and de Sitter solutions as critical points, e.g. attractor and repeller solutions, in terms of the scalar couplings. Many of these solutions have been overlooked in the literature.

We provide specific examples for double and triple exponential potentials with one and two scalars, where we find numerical solutions, which interpolate between the critical points. Some of these correspond to the reduction of new exotic S-brane solutions.

1 Introduction

The discovery that the universe might currently be in a phase of accelerate expansion [1, 2] has led to a large interest in finding de Sitter solutions or more general accelerating cosmologies from M-theory, see e.g. [3–15] and references therein.

A simple way to study accelerating cosmologies is to consider models containing just gravity and a number of scalars with a potential. This has a long history and has resulted in models for inflation [16], describing the early universe, and later for quintessence [17], where a scalar field gives rise to a small effective cosmological constant. Multi-exponential potentials comprise a specific class of potentials which have been studied a lot, and these are of interest for two reasons. First, they can arise from M-theory in many ways, e.g. via compactifications on product spaces possibly with fluxes [18–21], and second, the equations of motion can be written as an autonomous system. This approach allows for an algebraic determination of power-law and de Sitter solutions, i.e. critical points, which can correspond to early- and late-time asymptotics of general solutions. Many authors have made use of this fact, see e.g. [13, 22–28] and references therein.

The understanding of multi-exponential potentials has gradually evolved over the years. In the early days the single exponential was studied in the context of inflation, where it was discovered that this potential allowed for a power-law solution [22]. Later on the effect of additional exponential terms each carrying a different scalar was studied. This model is called "assisted inflation" [26]. The outcome is that the scalars 'assist' each other, in the sense that each term contributes in the same way to the power-law behaviour of the scale factor and all the contributions are added. Later on the effect of a cross coupling between scalars was searched for, this resulted in a model called "generalized assisted inflation" [29]. It was shown that these multi-exponential potentials also allowed for power-law solutions. However the understanding of multi-exponential potentials wasn't complete. The class of potentials described in [29] doesn't cover all the possible multi-exponential potentials. There was a strong restriction on the scalar couplings in their model, such that it only allows for power-law solutions, namely the potentials don't have any extrema. But, a considerable amount of models nowadays which are inspired by string theory seem to be multi-exponentials with extrema (which allow for de Sitter solutions). Hence they don't fall in the class of generalized assisted inflation.

The goal of this paper is to study the most general multi-exponential potential. This is done using the elegant formalism of autonomous dynamical systems. We construct all possible power-law solutions and de Sitter solutions by constructing the critical points to which they correspond in this formalism. We point out that even in the case of generalized assisted inflation many power-law solutions which correspond to so called non-proper critical points were overlooked. To illustrate this we consider the special cases of double and triple exponential potentials with one or two scalars. These can arise from M-theory, and the interpolating solutions then correspond to the reduction of S-branes [30] and so-called exotic S-branes [13], respectively. For the exotic S-branes we derive the phases of accelerate expansion and find special cases where the number of such phases can be arbitrarily high. This can be useful for solving the cosmological coincidence problem, since oscillating dark energy could give an explanation of why we see a recent take over of dark energy in our present universe. It would simply be an event that occurs for many times during the evolution of the universe.

The paper is organised as follows. In section 2 we present the system consisting of gravity and scalars with a potential. In section 3 we perform the general analysis of critical points. In section 4 we consider the special cases of double exponentials. In section 5 we present cases which can be obtained from the reduction over a three-dimensional group manifold. Finally, we end with a discussion of our results in section 6.

2 Scalar Gravity with Multi-exponential Potentials

We consider 4-dimensional spatially flat FLRW gravity with N scalars ϕ_I which only depend on (cosmic) time τ. The scalars have a potential which is of the most general exponential form:

$$V(\vec{\phi}) = \sum_{i=1}^{m} \Lambda_i \, e^{-\vec{\alpha}_i \cdot \vec{\phi}} \,. \tag{1}$$

Thus, the scalar potential is characterised by m vectors $\vec{\alpha}_i$ and m constants Λ_i which can have positive or negative signs. The $\vec{\alpha}_i$ vectors form an $m \times N$ matrix α_{iI}, where the indices $i = 1, \ldots, m$ parametrise the exponential terms in the potential and the indices $I = 1, \ldots, N$ parametrise the different scalars. The Lagrangian for the system then reads[1]:

$$\mathcal{L} = \sqrt{-g} \left(R - \tfrac{1}{2} (\partial \vec{\phi})^2 - V(\vec{\phi}) \right) . \tag{2}$$

The equations of motion derived from the Lagrangian are

$$\begin{aligned} &\ddot{\phi}_I + 3H\dot{\phi}_I + \frac{\partial V}{\partial \phi_I} = 0 \,, \\ &H^2 = \tfrac{1}{12} (\dot{\vec{\phi}} \cdot \dot{\vec{\phi}}) + \tfrac{1}{6} V \,, \\ &\dot{H} = -\tfrac{1}{4} (\dot{\vec{\phi}} \cdot \dot{\vec{\phi}}) \,, \end{aligned} \tag{3}$$

where the dot is differentiation w.r.t. cosmic time. We refer to the equations as the scalar equations, the Friedmann equation and the acceleration equation, respectively. The Hubble constant H is defined as $H = \dot{a}/a$ where $a(\tau)$ is the scale factor appearing in the flat FLRW metric:

$$ds^2 = -d\tau^2 + a(\tau)^2 \, dx_3^2 \,. \tag{4}$$

There are $N + 1$ degrees of freedom, namely, the scale factor and the N scalars (and accordingly only $N+1$ equations of motion are independent. For example, the acceleration equation can be obtained from the Friedmann equation and the scalar equations). There exist 2 types of solutions:

[1] We use the convention for the metric with signature mostly plus.

- Critical points: These solutions correspond to stationary solutions defined in terms of certain dimensionless variables, which will be introduced in the next section. The critical points can be obtained explicitly and they correspond to power-law solutions ($a(\tau) \sim \tau^p$) or de Sitter solutions[2] ($a(\tau) \sim e^\tau$). The solutions can be either attractors, repellers or saddle points. In the former two cases they correspond to the asymptotic behaviour of more general solutions, whereas a saddle point just corresponds to an intermediate regime.

- Interpolating solutions: These are the non-stationary solutions and in general they will interpolate between the critical points. Often they can not be found explicitly, but a numerical analysis can reveal most of their properties.

3 The Critical Points

Critical points (also known as fixed points or equilibrium points) are solutions of differential equations in the context of autonomous dynamical systems. An autonomous system is defined as a system described by n variables, say $\vec{z}$, whose dynamical equations are of the form:

$$\frac{d\vec{z}}{dt} = \vec{f}(\vec{z})\,, \tag{5}$$

where $\vec{f} : \mathbb{R}^n \to \mathbb{R}^n$ is interpreted as a vector field on $\mathbb{R}^n$. The differential equations then imply that the vector field $\vec{f}$ is everywhere tangent to the possible orbits. Critical points of an autonomous system are defined as those points $\vec{z}_0$ obeying $\vec{f}(\vec{z}_0) = 0$. These points are always exact constant solutions since $d\vec{z}_0(t)/dt = 0$. The nice property about these systems is that the critical points are often the end points (and initial points) of the orbits and therefore describe the asymptotic behaviour. If the solutions interpolate between critical points they can be divided into two classes:

- Heteroclinic orbit: This is an orbit connecting two different critical points.

- Homoclinic orbit: This is an orbit connecting a critical point to itself.

Most of the examples we found are of the first type and we will focus on these. More on the theory of dynamical systems in cosmology can be found in [31, 32].

A nice property of multi-scalar cosmology with exponential potentials is that they allow for a description in terms of variables that make the system autonomous [22, 24, 25, 28]. With an arbitrary multi-exponential potential, the variables are defined as follows:

$$x_I = \frac{\dot{\phi}_I}{\sqrt{12}\,H}\,, \quad y_i = \sqrt{\frac{\Lambda_i\, e^{-\vec{\alpha}_i\cdot\vec{\phi}}}{6H^2}}\,. \tag{6}$$

[2]Anti-de Sitter solutions are not possible since a flat FLRW metric doesn't support them

In this notation, there are $N+m$ variables. Note that y_i will be imaginary when $\Lambda_i < 0$, but this is not a problem since only y_i^2 appears in the equations of motion. Rewriting the equations of motion with these variables yields

$$\frac{\dot{x}_I}{H} = -3\,y^2\,x_I + \sqrt{3}\,\sum_{i=1}^{m} \alpha_{iI} y_i^2\,, \tag{7}$$

$$x^2 + y^2 = 1\,, \tag{8}$$

$$\frac{\dot{H}}{H^2} = -3\,x^2\,, \tag{9}$$

where we have used the shorthand notation $x^2 = \sum_{I=1}^{N} x_I^2$ and $y^2 = \sum_{i=1}^{m} y_i^2$. A nice consequence of the choice of variables is that the Friedmann equation (8) becomes the defining equation of an $(N+m-1)$-sphere (for $\Lambda_i > 0$, otherwise it will be a generalised hyperboloid). Furthermore, from the acceleration equation, it is easily seen that the condition for accelerate expansion is

$$\ddot{a} > 0 \qquad \Leftrightarrow \qquad x^2 < \frac{1}{3}\,. \tag{10}$$

The above condition allows us to visualise the region of acceleration for the specific examples in section 4 and 5.

It turns out that we also need the derivatives of the y-variables:

$$\frac{\dot{y}_i}{H} = \sqrt{3}\,(\sqrt{3}\,x^2 - \vec{\alpha}_i \cdot \vec{x})\,y_i\,. \tag{11}$$

We can also use $\ln(a)$ as evolution parameter[3] instead of cosmic time and it simplifies the equations since H drops out in the scalar equations of motion and the equations for $\dot{y}_i$, giving

$$\boxed{x_I' = -3\,y^2\,x_I + \sqrt{3}\,\sum_{i=1}^{m} \alpha_{iI} y_i^2\,, \quad y_i' = \sqrt{3}\,(\sqrt{3}\,x^2 - \vec{\alpha}_i \cdot \vec{x})\,y_i\,,} \tag{12}$$

where the prime indicates differentiation w.r.t. $\ln(a)$. The above is clearly of the form (5), and the critical points can therefore be calculated as $x_I' = y_i' = 0$ (or equivalently as $\dot{x}_I = \dot{y}_i = 0$). It is easy to prove that the system will obey the Friedmann constraint (8) at all times as long as it does so initially. So, if we impose (8) on the initial conditions then (12) contains all information of the subsequent evolution.

Integrating the acceleration equation (9) for a critical point yields power-law solutions if $x^2 \neq 0$

$$a(\tau) \sim \tau^p, \qquad p = \frac{1}{3\,x^2}\,. \tag{13}$$

[3] One has to be careful if the scale factor is not strictly monotonous.

If on the other hand $x^2 = 0$ we are in an extremum of the potential with a de Sitter expansion

$$a(\tau) \sim \exp(\sqrt{\tfrac{1}{6}V(\phi_c)}\,\tau)\,. \tag{14}$$

The equations (12) determining the critical points are

$$(\sqrt{3}\,x^2 - \vec{\alpha}_i \cdot \vec{x})\,y_i = 0\,, \tag{15}$$

$$-\,3\,y^2\,x_I + \sqrt{3}\,\sum_{i=1}^{m} \alpha_{iI} y_i^2 = 0\,. \tag{16}$$

There are two different kinds of critical points:

- The proper solutions: $\nexists i : y_i = 0\,,$
- Non-proper solutions: $\exists i : y_i = 0\,.$

We can single out special non-proper solutions, which always exist, namely the case where all y's vanish. From the Friedmann equation it follows that these solutions have $x^2 = 1$ and for this reason we refer to them as "the equator". The solutions with some y's vanishing have infinite scalars and are therefore not proper solutions of the equations of motion, but they are important as asymptotic behaviour of interpolating solutions. From this classification we see that there are a maximum of 2^m types of critical point solutions [28]. Below we will give these solutions for the most general exponential potential by analysing (15) and (16).

The rank R of the matrix α_{iI}, i.e. the number of independent $\vec{\alpha}_i$-vectors, plays a central role in this discussion. In fact, the discussion of the general potential naturally splits up into two cases: $R = m$ and $R < m$.

The rank R gives the effective number of scalars appearing in the potential, corresponding to the part of the scalar space that is projected on the $\vec{\alpha}_i$-vectors. It is therefore always possible to perform a field redefinition, such that only R scalars appear in the potential. The part of the scalar space perpendicular to the $\vec{\alpha}_i$-vectors only appears in the kinetic term of the Lagrangian and is $(N-R)$-dimensional. These scalars therefore decouple from the rest. All systems with $N > R$ have decoupled scalars and this is necessarily the case when $N > m$. Systems with $N \leq m$ only have decoupled scalars if the vectors $\vec{\alpha}_i$ are linearly dependent in such a way that $N > R$.

The field redefinition yielding R scalars in the potential can be performed by an $SO(N)$ rotation (which leaves the kinetic term invariant) such that $\vec{\phi}$ changes into $\vec{\phi}'$ and $\alpha'_{iR+1} = \alpha'_{iR+2} = \ldots = \alpha'_{iN} = 0$ for all i. We then notice from (16) that for critical points all x's corresponding to decoupled scalars are zero, $x_{R+1} = x_{R+2} = \ldots = x_N = 0$. So in the rest of this section, the indices I now run from 1 to R. In the case $R = m$, this makes α_{iI} a square matrix.

We have seen that the discussion of the system can be split up into two cases, depending on the rank of α_{iI}. Alternatively, we can formulate this in terms of the following matrix, which is quadratic in the α's

$$A_{ij} = \vec{\alpha}_i \cdot \vec{\alpha}_j \, . \tag{17}$$

The separation of the general exponential potential into two classes can then be characterised by the determinant of A:

$$\begin{aligned} R = m: \qquad & \det(A) \neq 0 \, , \\ R < m: \qquad & \det(A) = 0 \, . \end{aligned} \tag{18}$$

The first class corresponds to an invertible A-matrix and this is exactly what is termed generalized assisted inflation [29], whereas the second class, to our knowledge, has not been treated in generality in the literature.

We will extend the existing results by also treating the case of non-invertible A in generality and by giving also the non proper critical points of both classes. Special examples can be obtained by performing compactifications over certain three-dimensional unimodular group manifolds, corresponding to class A in the Bianchi classification see e.g. [13].

There is a subtlety about the description in terms of the (x_I, y_i)-variables, namely if $R < m$ then the y-variables are not necessarily independent. We will comment on this in section 3.2.

3.1 The $R = m$ Case

A nice feature about this case is that $\dot{x}_I = 0$ implies $\dot{y}_i = 0$. This can be seen in the following way: First we differentiate (7) and use $\dot{x}_I = d(y^2)/d\tau = 0$. Multiplying with α_{jI} and summing over I we get $\sum_j (A_{ij}) \, d(y_j^2)/d\tau = 0$, and since $\det(A) \neq 0$ we know that the only solution is $d(y_i^2)/d\tau = 0$.

We will now solve for the critical points:

- Proper critical points. From (15) and (16) we get:

$$\sum_i (A_{ij}) \, y_i^2 = 3y^2 \, x^2 \, e_j \, , \tag{19}$$

where e_j is an m-dimensional vector with all components equal to 1. Inverting this relation and using (16) yields the values of y_i and x_I for the proper critical point

$$y_i^2 = \frac{3p-1}{3p^2} \sum_{j=1}^{m} (A^{-1})_{ij} \, , \qquad x_I = \frac{\sqrt{3}\, p}{3p-1} \sum_i^{m} \alpha_{iI} \, y_i^2 \, , \tag{20}$$

where p is the exponent given in (13). The result for x_I can also be given in the rotated basis where α_{iI} is a square matrix

$$x_I = \frac{1}{\sqrt{3}\, p} \sum_{i=1}^{m} (\alpha^{-1})_{iI} \, . \tag{21}$$

Note that by construction the A_{ij}-matrix (17) is $SO(N)$-invariant and accordingly all quantities containing only this matrix can be calculated in any basis. We notice from our formula above that there is a unique proper critical point. However, it only exists when y_i^2, determined from (20), has the same sign as Λ_i, which serves as a consistency check of definition (6). Thus, this critical point only exists for certain values of the α-vectors.

Using (13) we get the exponent for the power-law, which reproduces the result found in [29, 33]:

$$p = \sum_{i,j=1}^{m} (A^{-1})_{ij} \,. \tag{22}$$

By integration we can go back to the ϕ_I, H variables where the solution becomes:

$$H = \frac{p}{\tau}\,, \qquad \phi_I = \sqrt{12}\, p\; x_I\; \ln(\tau) + c_I\,, \qquad y_i^2 = \frac{k_i}{\tau^2}\,, \tag{23}$$

where c_I and k_i are integration constants. In fact, in [21, 29], (23) was used as an Ansatz to find power-law solutions.

- Non-proper critical points. These correspond to some y's being equal to zero. Parametrising the subset of nonzero y's with the indices $a, b, c, \ldots$, the equations become:

$$\sqrt{3}x^2 - \vec{\alpha}_a \cdot \vec{x} = 0\,, \qquad \sum_a \alpha_{Ia}(y_a)^2 - (1 - x^2)\sqrt{3}x_I = 0\,, \tag{24}$$

from which we deduce:

$$\sum_b (A_{ab}) y_b^2 = 3\, y^2\, x^2\, e_a\,. \tag{25}$$

The $\vec{\alpha}_a$-vectors are of course also linearly independent and accordingly the sub-matrix A_{ab} has non-zero determinant and is invertible. Inverting relation (25) and using (16), we find a unique solution

$$y_a^2 = \frac{3p-1}{3p^2} \sum_b (A^{-1})_{ab}\,, \qquad x_I = \frac{\sqrt{3}\,p}{3p-1} \sum_a^m \alpha_{aI}\, y_a^2. \tag{26}$$

The power-law is again given by (22) but now with the inverse of the sub-matrix A_{ab}. Just as for the proper solution, the above is only well-defined when y_a^2 has the same sign as Λ_a. Note that all the above formulae for the non-proper critical points are similar to those for the proper ones. This is a consequence of the fact that vanishing y's just yield a truncated potential.

Note that, since the solution for the proper critical point is unique and has power-law behaviour for the scale factor, there are no de Sitter solutions. This can also be seen from (19). Since A has maximal rank, this matrix only has the trivial nullspace, i.e $y_i = 0$, which is not consistent with the Friedmann equation, since $x = 0$ for the de Sitter solutions. We can conclude that potentials with linearly independent $\vec{\alpha}_i$-vectors generically have power-law solutions and no de Sitter solutions. This conclusion was also reached in [34] where special cases were considered.

The special case where α_{iI} is diagonalisable by an $SO(N)$-rotation is equivalent to the case where just one scalar appears in each exponential, thus yielding the model which has been called assisted inflation [26].

3.2 The $R < m$ Case

Since $\det(A) = 0$ we will have to use another approach to determine the critical points. And also the $R < m$ case will be more difficult to treat in full generality because the y's are not necessarily independent.

The number of independent y's is always smaller than or equal to $R + 1$, as we will now illustrate. After possible field redefinitions, the y-coordinates are given in terms of $R + 1$ fields, namely the scalars and the Hubble parameter. Among the m coordinates we therefore at most have $R + 1$ independent, e.g. $y_1, \ldots, y_{R+1}$, and this leaves us with $m - R - 1$ relations for the rest of the y's. From the definitions of the y's, we can express ϕ_I and H in terms of the first $R + 1$ y's

$$e^{\phi_I} = \prod_{i=1}^{R} \left(\frac{y_i^2\, \Lambda_{i+1}}{y_{i+1}^2\, \Lambda_i} \right)^{(\beta^{-1})_{Ii}}, \qquad H = \frac{\Lambda_{R+1}}{6}\, e^{-\vec{\alpha}_{R+1}\cdot\vec{\phi}}\, y_{R+1}^{-2}\,, \tag{27}$$

where the following square matrix has been defined

$$\beta_{iJ} = \alpha_{i+1,J} - \alpha_{iJ}\,, \quad i, J \in \{1, \ldots, R\}\,. \tag{28}$$

We can then express the remaining y's in terms of the first $R + 1$ as follows

$$y_i^2 = y_{R+1}^2\, \frac{\Lambda_i}{\Lambda_{R+1}} \frac{\prod_{j,K=1}^{R} \left(\frac{y_j^2\, \Lambda_{j+1}}{y_{j+1}^2\, \Lambda_j} \right)^{\alpha_{iK}(\beta^{-1})_{Kj}}}{\prod_{l,M=1}^{R} \left(\frac{y_l^2\, \Lambda_{l+1}}{y_{l+1}^2\, \Lambda_l} \right)^{\alpha_{R+1,M}(\beta^{-1})_{Ml}}}\,, \qquad i = R+2, \ldots, m\,. \tag{29}$$

Thus, the maximal number of independent y's is $R + 1$. It is possible to prove that the above relations for the y_i's will be obeyed for the dynamical system (12) at all times if they do so initially. So again we can use (12) as equations which govern the whole system as long as we pick our initial conditions consistent. With this in mind we will look for critical points.

Until now we denoted the row vectors of the α-matrix with $\vec{\alpha}_i$ and A_{ij} is the matrix with as entries the inner products of these row vectors: $A_{ij} = \vec{\alpha}_i \cdot \vec{\alpha}_j$. In this section we will also need the column vectors which we will denote by $\vec{\alpha}_I$ and we then define the following matrix

$$B_{IJ} = \vec{\alpha}_I \cdot \vec{\alpha}_J\,. \tag{30}$$

The R column vectors $\vec{\alpha}_I$ are all linearly independent because the rank of α equals R and as a consequence B is invertible (remember that I now runs from 1 to R). It is this property that we will use to find the solutions.

- Proper power-law critical points. Looking for the solution(s), with $y_i \neq 0$, we find from (15)

$$\sum_{I=1}^{R} B_{IJ} x_I = \sqrt{3} x^2 F_J \,, \tag{31}$$

where $F_J = \sum_{i=1}^{m} \alpha_{iJ}$. Thus we can solve for x_I:

$$x_I = \frac{1}{\sqrt{3}\, p} \sum_{J=1}^{R} (B^{-1})_{IJ}\, F_J \,, \tag{32}$$

and hence we find the extension of the power-law formula to the case where $R < m$:

$$p = |B^{-1} \cdot \vec{F}|^2 \,. \tag{33}$$

One can prove that this formula reduces to (22) if $R = m$. Since the rank of α_{iI} is R, it is enough to use R independent equations among the m equations of (15) to obtain x_I. This result of course has to be consistent with the remaining $m - R$ equations, and this puts strong restrictions on the allowed dilaton couplings as we will now show. Let $\{\vec{\alpha}_a\}_{a=1}^{R}$ be linearly independent. It is possible to solve (15) simultaneously for these vectors. The rest of the vectors can be written as linear combinations and are only guaranteed to solve (15) if the linear combinations are convex [4]

$$\vec{\alpha}_i = \sum_{a=1}^{R} c_{ia} \vec{\alpha}_a \,, \qquad \sum_{a=1}^{R} c_{ia} = 1 \,, \qquad i = R+1, \ldots, m \,. \tag{34}$$

We will give a specific example, which has an M-theory origin, where this is realised. A special case is $R = 1$, where after field redefinitions only one scalar appears in the potential. In this case, the above solution will never exist, since (15) becomes m equations with one variable (or equivalently, the requirement of convexity here would imply $m = 1$ which is not the case under consideration).

An important difference from the previous case is the question of the uniqueness of the solution. We can not obtain the y-values with this procedure, and in particular we cannot determine whether they are unique. In fact, it is easy to give an example where they are not: When at least one $\Lambda_i < 0$ we have the following possibility, since A has non-trivial kernel

$$\begin{aligned} & y^2 = 0 \,, \quad y_i^2 \in \mathrm{Ker}(A) \,, \\ & x^2 = 1 \,, \quad \vec{\alpha}_i \cdot \vec{x} = \sqrt{3} \,, \quad \text{for} \quad y_i \neq 0 \,. \end{aligned} \tag{35}$$

[4] Of course there are many ways to number the vectors; it is enough to find one which obeys these relations.

In particular, this includes a proper critical point of the form (32) when all $y_i \neq 0$ and where furthermore

$$|B^{-1} \cdot \vec{F}|^2 = \frac{1}{3} . \tag{36}$$

- De Sitter solutions. It was seen in the previous subsection, that de Sitter solutions do not exist for $R = m$, because the matrix A then has a trivial kernel. In the present case, since A has a non-trivial kernel, and therefore a de Sitter solution is possible

$$x = 0 , \quad y^2 = 1 , \quad y_i^2 \in \mathrm{Ker}(A) . \tag{37}$$

Again, this solution is only well-defined when y_i^2 has the same sign as Λ_i. We can conclude that potentials with $R < m$ show the opposite behaviour of $R = m$ potentials. Here (proper) power-law solutions are rare (only possible for certain couplings (34)) whereas de Sitter solutions are quite generic. Again, a similar observation was made in [34], but for specific couplings (which did not allow power-law behaviour).

- Non-proper critical points. Looking for these solutions, we again put a subset of the y's to zero. This corresponds to some terms in the potential being absent and we can therefore analyse the new system as before but with a "truncated" potential. A subtlety is that whenever $R < m$ the y's are dependent on each other, and therefore only certain subsets of the y's can be zero at the same time.

The findings of this section are summarised in the table below. The asterisk in the lower left corner symbolises the fact that we have a truncated system which can belong to either of the two cases ($R < m$ or $R = m$).

	$R < m , \quad \det(A) = 0$	$R = m , \quad \det(A) \neq 0$
Proper	Power-law (convex combinations)	Power-law
	de Sitter	No de Sitter
Non-proper	$*$	Power-law
		No de Sitter

Table 1: *The critical points for multi-exponential potentials.*

As mentioned, the critical points give rise to the asymptotic behaviour of the general solutions. By performing a stability analysis it is possible to determine the nature of the critical points, i.e. whether they are attractors, repellers or saddle points. This can be done by linearising the system around the critical points, $\vec{x}' = \mathbf{M} \cdot \vec{x}$, and determining the eigenvalues of the matrix $\mathbf{M}$. If the real part of all eigenvalues is negative, the critical point is an attractor, if the real part of all eigenvalues is positive, the critical point is a repeller and in the mixed case it is a saddle point. It is easy to perform the stability analysis in the simple cases considered in the following sections and the result is confirmed by the interpolating solutions, which are calculated numerically.

4 Double and Triple Exponential Potentials

In this section we will consider some specific examples of double and triple exponential potentials with one or two scalars, i.e. $m = 2, 3$ and $N = 1, 2$. These examples serve as an illustration of the formal framework in the previous section.

As mentioned, the critical points give the asymptotic behaviour of more general solutions. In some cases it has been possible to obtain these solutions exactly. For single exponential potentials, this has been done for arbitrary dilaton couplings and the result can be pictured as straight lines in the space defined by the x's [13]. For double exponential potentials, exact solutions have been obtained for special values of the dilaton couplings, corresponding to the reduction of S-brane solutions to 4D, see e.g. [9,10,12] and references therein. Ideally, we would like to obtain exact results for the general case. However, this is a highly non-trivial task, and we therefore turn to numerical methods, which can still show the qualitative behaviour of the solutions. To this end, it is convenient to use $\ln(a)$ as a time parameter. For an eternally expanding universe where a increases from 0 to ∞, our time coordinate ranges from $-\infty$ to ∞.

In general, an S-brane can be obtained as a time-dependent solution to the following system containing gravity, an antisymmetric tensor and possibly a dilaton

$$S = \int d^{4+d}x \, \sqrt{-\hat{g}} \left(\hat{R} - \tfrac{1}{2} (\partial\hat{\phi})^2 - \tfrac{1}{2d!} \, e^{-b\hat{\phi}} \, \hat{F}_d^2 \right) , \tag{38}$$

where the hats indicate that the fields live in $4 + d$ dimensions and where the dilaton coupling for maximal supergravities is given by

$$b = \sqrt{\frac{14 - 2d}{d + 2}} \, . \tag{39}$$

Reducing over a d-dimensional maximally symmetric space with curvature k and flux f yields the following potential [35]

$$V(\phi, \varphi) = f^2 \, e^{-b\,\phi - 3\sqrt{\frac{d}{d+2}}\,\varphi} - k \, e^{-\sqrt{\frac{d+2}{d}}\,\varphi} \, , \tag{40}$$

where φ is the Kaluza-Klein scalar. S2-brane solutions have been found in six to eleven dimensions, corresponding to $d = 2, \ldots, 7$. In five dimensions, an S2-brane has a 1-form field strength. The corresponding four-dimensional cosmological solution with single exponential potential was found in [13]. As explained in that paper, a general twisted reduction leads to triple exponential potentials, which could have corresponding exotic S-brane solutions in five dimensions.

4.1 Double Exponential Potentials, one Scalar

The simplest case is $m = 2$ and $N = 1$. The corresponding potential is then described in terms of two dilaton couplings α_1 and α_2. We can always choose e.g. α_1 to be positive and in this example we will start by considering positive Λ_i. Since $R = 1$, we have 2 independent y's. The Friedmann equation defines a 2-sphere, but the allowed solutions can only lie on the part corresponding to non-negative y's. Using the machinery from the previous section, we find the following critical points

$$
\begin{array}{llr}
(i) & y_1 = y_2 = 0\,, \quad x^2 = 1\,, & \\
(ii) & y_1 = \sqrt{1-\frac{\alpha_1^2}{3}}, \quad y_2 = 0\,, \quad x = \frac{\alpha_1}{\sqrt{3}}\,, \qquad \text{for} \quad \alpha_1^2 < 3 & \\
(iii) & y_1 = 0\,, \quad y_2 = \sqrt{1-\frac{\alpha_2^2}{3}}, \quad x = \frac{\alpha_2}{\sqrt{3}}\,, \qquad \text{for} \quad \alpha_2^2 < 3 & (41)\\
(iv) & y_1 = (1-\frac{\alpha_1}{\alpha_2})^{-1/2}\,, \quad y_2 = (1-\frac{\alpha_2}{\alpha_1})^{-1/2}\,, \quad x = 0\,, \quad \text{for} \quad \alpha_2 < 0\,. &
\end{array}
$$

The first corresponds to the "equatorial" points $x = \pm 1$. In an (y_1, y_2, x)-plot these become the North and South Pole. The proper solution only exists for $\alpha_2 < 0$ and then corresponds to a de Sitter solution. The stability of the different points is best illustrated by considering the different possible cases[5].

- $\alpha_1\,, \alpha_2 > \sqrt{3}$: Only the North and South Pole are critical Points; the former is attracting and the latter is repelling and any interpolating solution will be a curve in between, and these can be found numerically. An example is illustrated in the left part of figure 1.
- $\alpha_1 < \alpha_2 < \sqrt{3}$: The critical points (i)-(iii) exist. The poles are repellers and (iii) is attracting.
- $\alpha_1 < \sqrt{3}\,, \alpha_2 < -\sqrt{3}$: Apart from the poles, we have the two critical points, corresponding to (iii) and (iv) in (41). The North Pole is repelling and the de Sitter solution is attracting; this is shown on the right in figure 1.
- $\alpha_1 > \sqrt{3}\,, -\sqrt{3} < \alpha_2 < 0$: This is similar to the previous case, except that critical point (iii) is interchanged with (ii), and the early asymptotics will be the South Pole.
- $\alpha_1 > \sqrt{3}\,, 0 < \alpha_2 < \sqrt{3}$: In addition to the North and South Pole, there is the non-proper critical point (ii), which is an attractor. The South Pole is repelling. An interpolating solution is shown in the left part of figure 2.
- $\alpha_1 > \sqrt{3}\,, \alpha_2 < -\sqrt{3}$: The critical points are the poles and the de Sitter solution, and the latter is an attractor. It turns out that the poles are saddle points, and hence they do not give rise to the early asymptotics of the solution. Instead this will be an infinite cycle, moving closer and closer to the boundary of the space (given by $y_1 = 0$ or $y_2 = 0$) as time goes to minus infinity. This is illustrated to the right in figure 2.

[5] If we do not explicitly classify the stability of a critical point, it will be a saddle point.

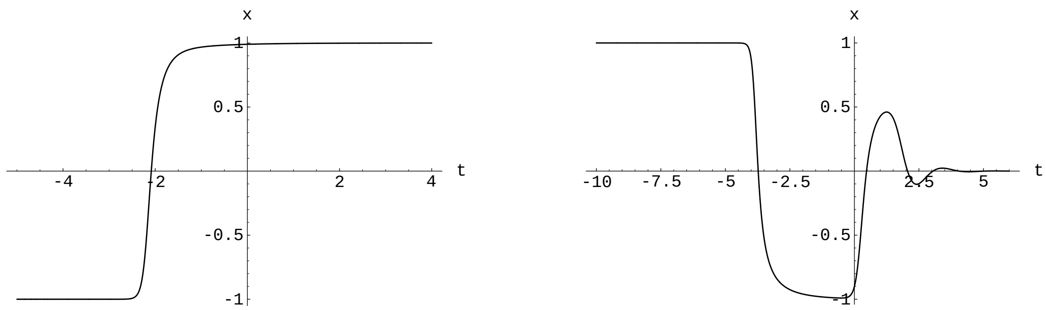

Figure 1: *The plot to the left shows $x(t)$ in the case $(\alpha_1, \alpha_2) = (3, 2)$, where the solution interpolates between the North and South Pole. The plot to the right is for the case $(\alpha_1, \alpha_2) = (1, -2)$, yielding a solution interpolating between the North Pole and a de Sitter solution.*

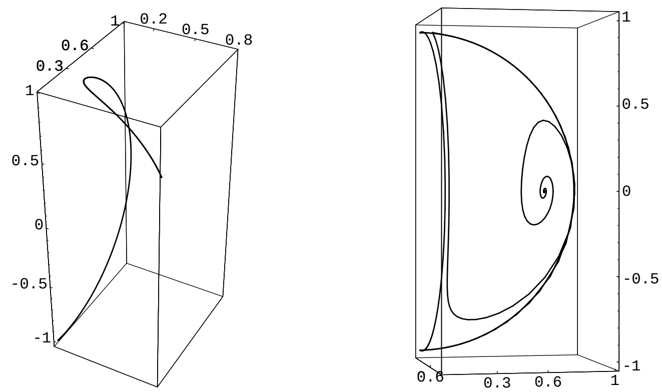

Figure 2: *The figure shows (y_1, y_2, x)-plots for two cases. The left part, with $(\alpha_1, \alpha_2) = (2, 1)$, shows a solution interpolating between the South Pole and the critical point (ii). The right plot, with $(\alpha_1, \alpha_2) = (3, -2)$ shows a solution spiralling towards the de Sitter point.*

- $\alpha_1 < \sqrt{3}\,, -\sqrt{3} < \alpha_2 < 0$: The late-time asymptotics are similar to the previous case. The early-time asymptotics are different due to the fact that all the critical points (i)-(iv) are realised. Both of the poles will be repelling, and depending on initial conditions either of these can give rise to the early-time asymptotics.

For all the cases above, the solutions enter a phase of acceleration when $x^2 < 1/3$. The cases with $|\alpha_1|\,, |\alpha_2| > 1$ give rise to one period of transient acceleration, and otherwise the solution will end up in a phase of eternal acceleration, which as mentioned is an asymptotic de Sitter phase when $\alpha_2 < 0$. In the case $\alpha_1 > \sqrt{3}\,, \alpha_2 < -\sqrt{3}$ the phase of late-time acceleration is preceded by an infinite cycle, alternating between acceleration and deceleration.

The case with $\Lambda_2 < 0$ can be analysed in a similar way, but the interpolating solutions will now be given by curves on a hyperboloid. The critical point (iii) will now only exist for $\alpha_2^2 > 3$, since this yields $y_2^2 < 0$. By the same token, the de Sitter critical point (iv)

only exist for $\alpha_2 > \alpha_1 > 0$.

The S-brane case, corresponding to $\hat{\phi} = 0$, gives the following dilaton couplings

$$\alpha_1 = 3\sqrt{\frac{d}{d+2}}\,, \quad \alpha_2 = \sqrt{\frac{d+2}{d}}\,. \tag{42}$$

This system, which can be obtained from eleven dimensions where it will give rise to SM2-brane solutions, was analysed in [14], where curvature of the external space is also included. It is seen that only the critical points (i) and (iii) exist for $\Lambda_2 > 0$ and (i) and (ii) exist for $\Lambda_2 < 0$, with the latter being attracting. In the latter case, we also have a de Sitter critical point which, however, is not an attractor.

4.2 Double Exponential Potential, two Scalars

There is another case with double exponential potentials which has two scalars, i.e. $m = 2$ and $N = 2$. Considering the two α-vectors to be independent, we get $R = 2$. The critical points can be obtained as a special case of the general analysis from the previous section and consist of the equator, $x^2 = 1$, $y_i = 0$, the proper critical point, $y_i \neq 0$ and two non-proper critical points with $y_i = 0\,, y_j \neq 0$, $i \neq j$.

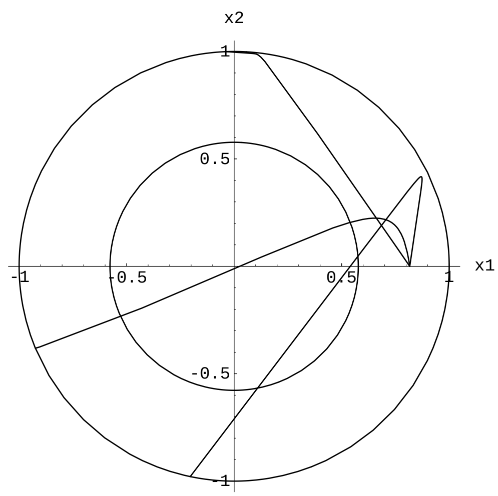

Figure 3: *Three interpolating solutions, corresponding to S2-branes reduced to four dimensions, projected on the (x_1, x_2)-plane. The inner circle is the boundary of the accelerating region.*

Specialising to the reduction of S-branes, we get the dilaton couplings

$$\vec{\alpha}_1 = (3\sqrt{\frac{d}{d+2}}, \sqrt{\frac{14-2d}{d+2}})\,, \quad \vec{\alpha}_2 = (\sqrt{\frac{d+2}{d}}, 0)\,. \tag{43}$$

For these α-couplings we get $\det(A) = 14/d - 2$, and therefore the matrix A is invertible for $d < 7$. However, using (20), y_1^2 is seen to be negative and since $\Lambda_1 = f^2 > 0$, the proper critical point does not exist. Apart from the equator, there is another critical point, which has $y_1 = 0$ and $y_2 \neq 0$ and corresponds to a power-law behaviour with exponent $p = d/(d+2)$. This critical point will be an attractor and the equator will be a repeller. Thus, an S2-brane reduced to four dimensions corresponds to a solution interpolating between the equator and the attracting critical point. This is similar to the behaviour of the solution found in [8], which is the fluxless limit of a reduced S2-brane [9]. Indeed, the attracting power-law solution is the same with or without flux. Examples of interpolating solutions, projected on the (x_1, x_2)-plane, are shown in figure 3 in the case of $d = 2$. It is seen that they indeed interpolate between the equator and the attracting critical point, which according to (26) has the coordinates $(x_1, x_2) = (\sqrt{2/3}, 0)$. For $d = 7$ there is a possibility of a de Sitter solution since $\det(A) = 0$, see (37), but again it does not exist because y_1^2 is negative.

4.3 Triple Exponential Potential, one Scalar

This example is the simplest case where the y-variables are not all independent and this subsection serves as an illustration. The potential is described in terms of three dilaton couplings α_1, α_2 and α_3. For simplicity, we take $\Lambda_i > 0$; the case with negative Λ_i can be analysed in a similar way. We can choose $\alpha_3 > 0$. Since $R = 1$, we have two independent y's, leaving one relation, which reads

$$(\Lambda_2)^{\alpha_1-\alpha_3}\,(y_2^2)^{\alpha_3-\alpha_1} = (\Lambda_1)^{\alpha_2-\alpha_3}\,(\Lambda_3)^{\alpha_1-\alpha_2}\,(y_1^2)^{\alpha_3-\alpha_2}\,(y_3^2)^{\alpha_2-\alpha_1}\,. \tag{44}$$

The analysis of critical points is analogous to the previous case, except for the extra feature of the relation above. There are three kinds of critical points (for the moment we forget about the y-dependence)

$$\begin{aligned}
&(i) \qquad y_i = 0\,, \quad x^2 = 1\,,\\
&(ii) \qquad y_i = y_j = 0\,,\ y_k = \sqrt{1-\frac{\alpha_k^2}{3}}\,,\ x = \frac{\alpha_k}{\sqrt{3}}\,, \quad i,j,k \quad \text{different}\\
&(iii) \qquad x = 0\,.
\end{aligned} \tag{45}$$

A necessary condition for its existence is $\alpha_k^2 < 3$. However, this is not sufficient, since (44) only allows certain y's to be non-zero while the others are zero. For instance, with $\alpha_3 > \alpha_2 > \alpha_1$, having $y_1 = 0$ or $y_3 = 0$ implies $y_2 = 0$.

The third type of critical point is a de Sitter solution given by the following equations

$$\alpha_1\, y_1^2 + \alpha_2\, y_2^2 + \alpha_3\, y_3^2 = 0\,, \qquad y_1^2 + y_2^2 + y_3^2 = 1\,, \tag{46}$$

which can be rewritten as

$$\frac{\alpha_3-\alpha_1}{\alpha_3}\, y_1^2 + \frac{\alpha_3-\alpha_2}{\alpha_3}\, y_2^2 = 1\,, \tag{47}$$

which defines an ellipse for $\alpha_3 > \alpha_1, \alpha_2$. When substituting y_3, (44) also gives a curve in the (y_1, y_2)-plane, and the critical point is given as the intersection between these two curves.[6] As an example, for $(\alpha_1, \alpha_2, \alpha_3) = (-1/2, 1/2, 3/2)$ and $\Lambda_i = 1$, the de Sitter critical point becomes $(y_1 = 0.78, y_2 = 0.52, y_3 = 0.34)$. Figure 4 shows the time-development of an interpolating solution for this case. It is seen that the late-time behaviour indeed corresponds to the de Sitter critical point above.

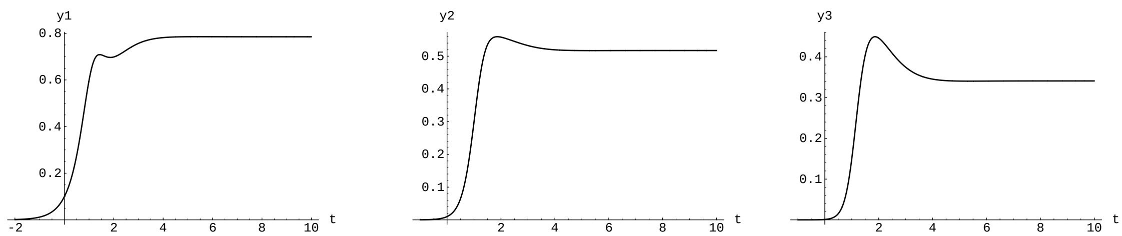

Figure 4: *The plots show $y_1(t)$, $y_2(t)$ and $y_3(t)$, respectively. As t increases, they tend towards the de Sitter critical point.*

5 Multi-exponential Potentials from Group Manifolds

In this section we will consider specific cases, which can be obtained by reducing pure gravity in seven dimensions over a three-dimensional group manifold. Since pure gravity in 7D can be embedded in 11D, the solutions have an M-theory origin. We will focus on the triple-exponential case.

Double exponential potentials can be obtained for certain truncations of reductions over type VIII and IX group manifolds [13]. This is equivalent to a trivial reduction over a circle followed by a reduction over a maximally symmetric 2D space with flux. The resulting potential is given by (40) with $d = 2$, and interpolating solutions correspond to reductions of S2-branes from six dimensions.

A triple exponential potential can be obtained from type VI_0 and VII_0 group manifolds and the result is [13]

$$V = \tfrac{1}{8}\, e^{-\sqrt{3}\varphi}\, (e^{\phi} \pm e^{-\phi})^2\,, \tag{48}$$

where the plus sign occurs for type VI_0 and the minus sign for type VII_0. We therefore have an example with $m = 3$ and $N = 2$, and the three dilaton couplings are

$$\vec{\alpha}_1 = (\sqrt{3}, 2)\,, \qquad \vec{\alpha}_2 = (\sqrt{3}, -2)\,, \qquad \vec{\alpha}_3 = (\sqrt{3}, 0)\,. \tag{49}$$

Note that only two of these are independent, and this case therefore falls into the class with $R < m$, and more interestingly, we find the convex combination $\frac{1}{2}\vec{\alpha}_1 + \frac{1}{2}\vec{\alpha}_2 = \vec{\alpha}_3$, so

[6]However, it is only possible to give algebraic expressions of the solution for special values of the dilaton couplings.

there is a possibility of a proper critical point with power-law behaviour. The fact that we have linearly dependent $\vec{\alpha}_i$-vectors ($R < m$) is actually the case for most class A Bianchi types. For the present example, there will be two independent y-variables plus the relation $y_3 = \pm 2\, y_1 y_2$, but only y_1 and y_2 are needed.

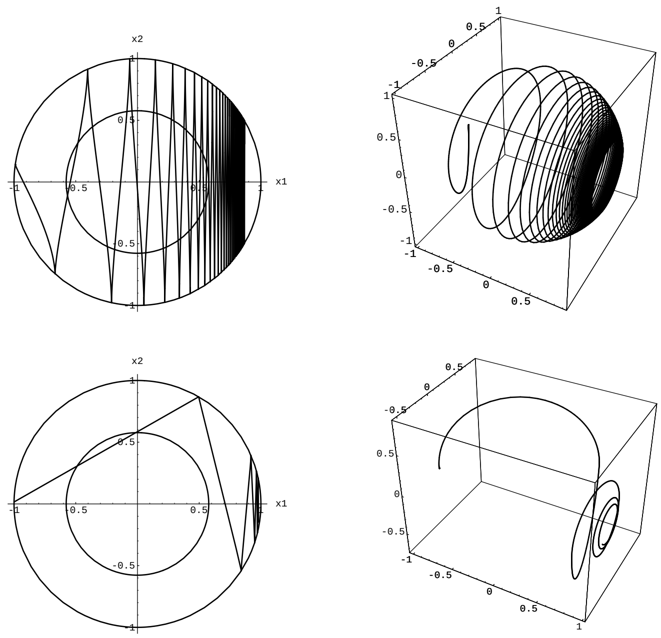

Figure 5: *Type VII$_0$. To the left are shown the projection of two interpolating solutions on the (x_1, x_2)-plane. To the right, the curves are shown on the 2-sphere defined by the Friedmann equation; the vertical axis is given by $y_1 - y_2$. The fact that the curves do not reach the attractor is due to the finite computation time.*

For the sake of illustration, $y_1 \pm y_2$ can be used as a variable, such that any solution will be given by points or curves on a 2-sphere (the upper half in case of the plus sign, since the y's are positive). Interestingly, the dilaton couplings are such that most of the critical points from the previous section do not exist. In fact, for type VI$_0$ we are just left with the equator solutions and for type VII$_0$ we have the equator and an infinite set of proper solutions

$$\begin{array}{rll} x^2 = 1\,, & y_1 = y_2 = 0 & \text{type} \quad \mathrm{VI}_0\,, \\ x^2 = 1\,, & y_1 = y_2 = 0 & \multirow{2}{*}{$\left.\begin{array}{l}\\ \\ \end{array}\right\}$ \quad \text{type} \quad \mathrm{VII}_0\,. \\ (x_1, x_2) = (1, 0)\,, & y_1 = y_2 & \end{array} \tag{50}$$

By studying the derivatives of the coordinates, it can be shown that the following points are attractors

$$\begin{array}{lll} (x_1, x_2) = (1, 0) & y_1 = y_2 = 0 & \text{type} \quad \mathrm{VI}_0\,, \\ (x_1, x_2) = (1, 0) & y_1 = y_2 & \text{type} \quad \mathrm{VII}_0\,. \end{array} \tag{51}$$

Thus, the latter is not unique since the y_i-values are arbitrary. The solution corresponds to (32), which is possible because of the convex combination: $\vec{\alpha}_3 = (\vec{\alpha}_1 + \vec{\alpha}_2)/2$. In both cases any interpolating solution will end in the point $(1,0,0)$ on the 2-sphere. In case VII_0, the y-value will be determined by the initial conditions. The sign of $\dot{x}_1$ is always positive. When projected on the (x_1, x_2)-plane, any curve will therefore move from left to right.

A stability analysis leads to the result that only the part of the equator with $x_1 < -1/7$ is repelling. Thus, any interpolating solution can start on this part and will end in $(1,0,0)$.

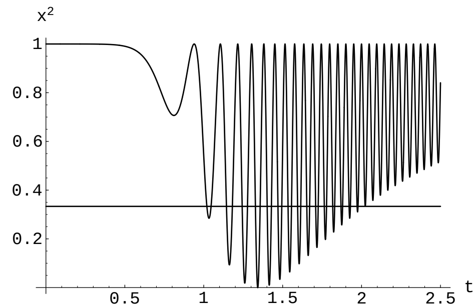

Figure 6: *Type VII*$_0$*. An example of a* (t, x^2)*-plot with* $n < 1$*.*

A couple of typical curves for interpolating solutions with different initial conditions are depicted on figure 5 for type VII_0. In this case, any curve will be spiralling around the 2-sphere towards the attractor. The projection on the (x_1, x_2)-plane produces a curve which bounces off the boundary of the unit circle. The inner disc, corresponding to $x^2 < 1/3$, yields phases of accelerate expansion. Depending on the initial conditions, the number of such phases can be as high or low as we want. For the two cases on figure 5, the numbers are 16 and 1, respectively. Even with a lot accelerating phases, the number of e-foldings is of order 1, and therefore these models are not well suited for inflation. The numerical solutions use $t = \ln(a)$ as time parameter. The number of e-foldings is

$$n = \ln\left(\frac{a(\tau_2)}{a(\tau_1)}\right) = \ln\left(\frac{e^{t_2}}{e^{t_1}}\right) = \Delta t\,, \tag{52}$$

and its order of magnitude can easily be read off from a (t, x^2)-plot as the sum of the t-intervals where $x^2 < 1/3$. An example is given in figure 6.

For type VI_0, the situation is slightly different, since $y_1 + y_2$ is always positive; this confines the curves to the upper half of the 2-sphere. As seen on figure 7, the curves will still move towards the attractor in an oscillatory manner, but now without crossing the equator (though they can get arbitrarily close). For this case, there can only be one or no phase of accelerate expansion.

The interpolating solutions above correspond to reductions of exotic S2-branes in five dimensions, or equivalently, exotic S$(D-3)$-branes in D dimensions. However, the solutions were found numerically, and we have not been able to obtain exact expressions for these exotic S-branes.

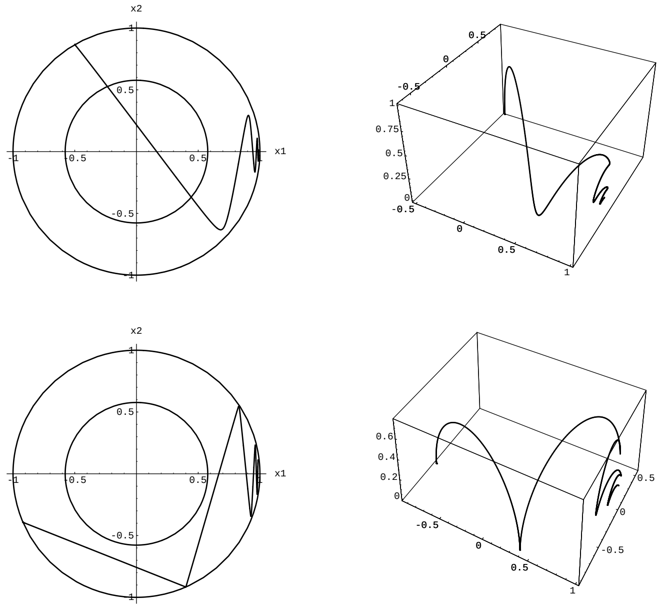

Figure 7: *Type* VI_0*. To the left are shown the projection of two interpolating solutions on the* (x_1, x_2)*-plane. To the right, the curves are shown on the 2-sphere defined by the Friedmann equation; the vertical axis is given by* $y_1 + y_2$*.*

6 Discussion

In this paper we have considered cosmological models for an arbitrary number of scalars with an arbitrary multi-exponential potential. Using a special set of variables, the equations were written as an autonomous dynamical system, and this allowed us to determine the critical points in complete generality. We found that the nature of these critical points depends strongly on the rank R of the matrix α_{iI}. The rank also determines the number of decoupled scalars.

In the case $R = m$, both the proper and non-proper critical points are power-law solutions, and there are no de Sitter solutions. In the $R < m$ case the opposite behaviour was found. Proper power-law solutions are only possible in special cases, where the $\vec{\alpha}_i$-vectors are linearly dependent in a specific way, but the de Sitter solutions are very generic. For the non-proper solutions this depends on whether the "truncated" potential has $R = m$ or $R < m$. We also found a new property of these systems, namely the possibility of proper critical points which are not unique. A special case was realised in section 5, where we have an infinite set of these.

We would like to emphasise that the non-proper critical points are as important as the proper solutions in understanding the interpolating solutions, even though they have not been considered often in the literature. In this respect, using the techniques of autonomous systems is more fruitful than simply looking for power-law solutions to the equations of motion.

It should be pointed out that our solutions generically have run-away behaviour of the scalars. The only exceptions are the de Sitter critical points, since these correspond to an extremum of the potential and accordingly stabilise the values of the scalars. This is important in the context of spontaneous decompactification [36] or stabilisation of dilaton and volume moduli [5]

In section 4 and 5 we gave several examples of double and triple exponential potentials. We presented the critical points and illustrated the interpolating solutions using numerical calculations. In particular, we found examples with an arbitrarily high number of phases of accelerate expansion. However, the number of e-foldings turns out to be of order one, so these models do not seem to be relevant for inflation. On the other hand, they might be relevant for present-day acceleration and more specifically for solving the cosmic coincidence problem.

The numerical solutions found in section 5 for the systems obtained from reductions over group manifolds of type VI_0 and type VII_0 correspond to the reduction of exotic S2-branes in five dimensions. The two solutions belong to a set of three different solutions, which can be obtained via twisted circle reductions. The third solution can be obtained from a reduction over the type II group manifold and corresponds to the reduction of a fluxless S2-brane. The existence of three classes of S-branes is similar to the cases of 7-branes in ten dimensions [37] and non-extremal D-instantons [38] and is reminiscent of the global $SL(2,\mathbb{R})$-symmetry of the higher dimensional theory. It would be interesting to see whether it would be possible to find exact solutions for the exotic S-branes.

Recently, an elegant framework for arbitrary potentials has been developed, where the solutions correspond to geodesics in an augmented target space [39]. One of the key ingredients is the importance of systems whose late-time behaviour is governed by single exponential potentials [12]. In our analysis these solutions asymptote to the special class of non-proper critical points where all y's but one vanish. However, we have shown that multi-exponential potentials have solutions, where the asymptotics can not governed by a single term in the potential. Specific examples are given by the cases of assisted inflation [26] and generalized assisted inflation [29] where each term in the potential contributes.

We would like to comment on some possible extensions of this work. First of all, we have only considered flat universes, and it is certainly possible to extend to the spatially curved cases. Secondly, we could also add matter,in the form of a barotropic fluid. This has been done in the case of assisted inflation in [24]. In the same spirit we hope to extend this to the most general exponential potential and report on it in a future publication. Thirdly, we could consider non-flat scalar manifolds. Finally, we could consider other specific numerical examples with other values of m and N and special dilaton couplings which arise from dimensional reductions of string/M-theory.

Acknowledgements

We thank Eric Bergshoeff, Martijn Eenink, Diederik Roest and Ulf Gran for interesting discussions. The work of Thomas Van Riet is part of the research programme of the "Stichting voor Fundamenteel Onderzoek van de Materie" (FOM). This work is supported in part by the European Community's Human Potential Programme under contract HPRN-CT-2000-00131 Quantum Spacetime, in which A.C., M.N. and T.V.R. are associated to Utrecht University.

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